Thedem 1:-

v. Imp *** Generating function:-

Show that $P_n(x)$ is the coefficent of h^n in the Expansion ~~of~~ in ascending powers of h of $[1-2hx+h^2]^{-1/2}$

(or)

Prove that $\sum_{n=0}^{\infty} h^n P_n(x) = [1-2xh+h^2]^{-1/2}$

Sol: $[1-2xh+h^2]^{-1/2} = [1-h(2x-h)]^{-1/2}$

$= 1+\frac{1}{2}h(2x-h)+\frac{1}{2}\frac{3}{4}h^2(2x+h)^2+\frac{1}{2}\frac{3}{4}\frac{5}{6}$

$$h^3(2x+h)^3 + \cdots + \frac{1\cdot3\cdot5\cdots(2n-3)}{2\cdot4\cdots(2n-3)} h^{n-1}(2x-h)^{n-1} + \frac{1\cdot3\cdot5\cdots(2n-1)}{2\cdot4\cdots(2n)} h^n (2x+h)^n + \cdots$$

Coeffecent of h^n $= \frac{1\cdot3\cdot5\cdots(2n-1)}{2\cdot4\cdots2n}(2x)^n + \frac{1\cdot3\cdot5(2n-3)}{2\cdot4\cdots(2n-2)}$

$$(2x)^{n-2}(-1)\,{}^{n-1}C_1 + \frac{1\cdot3\cdot5\cdots(2n-5)}{2\cdot4\cdots(2n-4)}$$

$$(2x)^{n-4}(-1)^2\,{}^{n-2}C_2 - \cdots$$

$$= \frac{1\cdot3\cdot5\cdots(2n-1)}{\cancel{2^n}\lfloor n}\cancel{2^n}x^n - \frac{1\cdot3\cdot5\cdots(2n-3)(2n-1)}{2\cdot4\cdots(2n-2)2n}$$

$$\left(\frac{2n}{2n-1}\right)2^{n-1}x^{n-2}\,n-1 + \frac{1\cdot3\cdot5\cdots(2n-5)(2n-3)(2n-1)}{2\cdot4\cdots(2n-4)(2n-2)(2n)}$$

$$\times\frac{(2n-2)(2n)}{(2n-3)(2n-1)}2^{n-4}x^{n-4}\frac{(n-2)(n-3)}{2\times1}\cdots$$

$$= \frac{1\cdot3\cdot5\cdots(2n-1)}{\lfloor n}\left[x^n - \frac{n(n-1)}{2(2n-1)}x^{n-2} + \frac{n(n-1)(n-2)(n-3)}{2\cdot4(2n-1)(2n-3)}x^{n-4} - \cdots\right]$$

$$= P_n(x)$$

$$\cancel{\sum_{n=0}^{\infty} h^n P_n(x) = (1-2xh+h^2)^{-1/2}}$$

$$\therefore \left[1-2xh+h^2\right]^{-1/2} = \sum_{n=0}^{\infty} P_n(x)h^n$$